ME-221
PROBLEM SET 1

Problem 1

Consider a system with input $u(t)$ and output $y(t)$ defined as

$$\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = u(t)$$

where $y(0) = \dot{y}(0) = 0$.

a) Show that this system is linear.
b) Propose a modification to the system so that it becomes non-linear.
b) Comment on the linearity of the system for non-zero initial conditions.

Problem 2

Consider a system with input $u(t)$ and output $y(t)$ defined as

$$y(t) = \int_{-\infty}^{2t} u(\tau)d\tau$$

Show that this system is time-variant.

Problem 3

Consider a system with input $u(t)$ and output $y(t)$ defined as

$$\dot{y}(t) = -2(t+1)y(t) + 3u(t)$$

a) Investigate the linearity, time-invariance, and causality of this system.
b) Propose a modification of the differential equation to generate a non-causal version of the system.

Problem 4

The sketch of two input signals, $u_1(t)$ and $u_2(t)$, are shown below. Given that $y_1(t)$ is the corresponding output of a LTI (linear and time-invariant) system when $u_1(t)$ is applied as the input, write the expression for the output $y_2(t)$ in terms of $y_1(t)$, where $y_2(t)$ is the output of the same LTI system to the input $u_2(t)$.

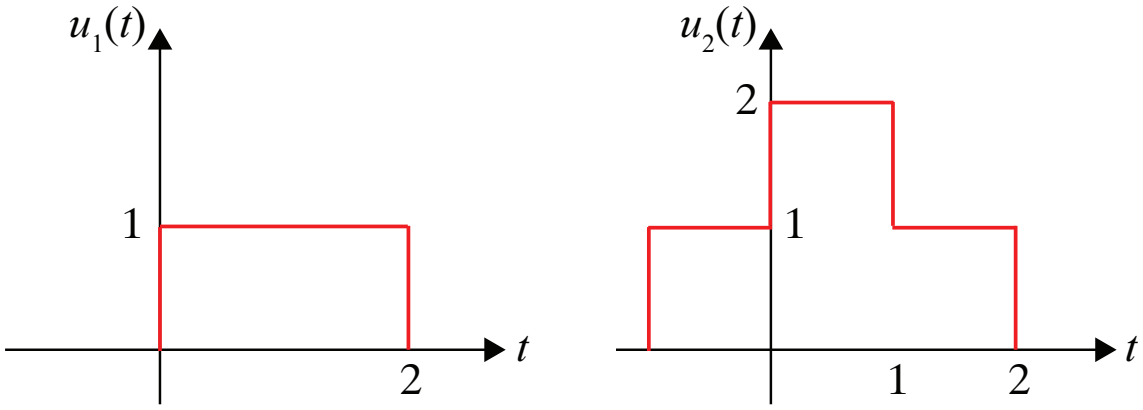

Figure 1: Signals used as an input for the LTI system.